

文章编号 1004-924X(2009)06-1228-05

# 微加速度计中 Sigma Delta 调制与 脉冲编码调制的对比分析

谭晓昀, 周贤中, 姜一鸣, 陈伟平

(哈尔滨工业大学 MEMS 中心, 黑龙江 哈尔滨 150001)

**摘要:**对 Sigma Delta 调制与脉冲编码调制进行对比,研究了 Sigma Delta 调制与脉冲编码调制的优缺点。建立了微加速度计中两位输出的 Sigma Delta 调制和两位输出的脉冲编码调制的仿真系统。然后,对比使用这两种调制方式的微加速度计的加速度信号,并分析在两种不同调制方式下的输出信号波形的差别。最后,利用周期图法得到了两种调制信号的功率谱密度图,发现在低频段脉冲编码调制信号的量化噪声比 Sigma Delta 调制信号大很多;而在高频段,两种调制方式信号的量化噪声一样严重。仿真结果表明,尽管 Sigma Delta 调制系统比脉冲编码调制系统更加复杂,但是 Sigma Delta 调制得到的信噪比要比脉冲编码调制得到的信噪比高出 20 dB。

**关键词:** Sigma Delta 调制; 脉冲编码调制; 微加速度计

**中图分类号:** TH824.4 **文献标识码:** A

## Comparative analysis of Sigma Delta modulation and pulse code modulation in micro-accelerometer

TAN Xiao-yun, ZHOU Xian-zhong, JIANG Yi-ming, CHEN Wei-ping

(MEMS Center, Harbin Institute of Technology, Harbin 150001, China)

**Abstract:** A comparative study of the Sigma Delta Modulation (SDM) and the Pulse Code Modulation (PCM) is presented, and the advantages and the disadvantages of the two kinds of modulation are also obtained. A 2-bit SDM simulated system and a 2-bit (PCM) simulated system for a micro-accelerometer are established in this paper. Then, the acceleration signals pass through the modulations are comparatively analyzed, and the difference of two modulations output waves is obtained by the comparative analysis. Finally, the power spectral density patterns of two kinds of the modulated signals are computed with a periodogram respectively. The power spectral density from experimental results indicates that the quantization noise of the PCM is more serious than that of the SDM in the low frequency, but the quantization noises of both modulation methods are the same in the high frequency. The simulated results show that the SNR of the SDM is 20 dB higher than that of the PCM, though the SDM modulation system is more complex than the PCM.

**Key words:** Sigma Delta modulation; pulse code modulation; micro-accelerometer

**Received date:** 2009-01-20; **Revised date:** 2009-04-30.

**Foundation item:** Supported by the National High-Tech Research and Development Program of China (863 Program)  
(Grant No. 2008AA042201)

## 1 Introduction

Recently, high performance micro-accelerometers have attracted more and more attention for their portable applications such as laptop computers and cellular phones. Thereby an analog-to-digital converter must be used as an interface to move the border between the analog and digital parts. Nowadays, Sigma Delta modulators are widely used in micro-accelerometers as interface circuits<sup>[1]</sup>. Micromachined accelerometers are one of the most important classes of MEMS devices that hold the second largest sales capacity after the pressure sensors. The applications of the inertial guidance, space microgravity, earthquake prediction, geophysical sensing and so on have promoted have led the demand for high performance micromachined inertial sensors, and lead to the high accuracy accelerometers to have a resolution of micro-*g*, even submicro-*g*. High performance inertial sensors usually have the advantages of the closed-loop control strategy increase the Dynamic Ranges (DR), linearity, and the bandwidth. To avoid the electrostatic pull-in problems in purely analog force-feedback closed-loop control strategies, the Sigma Delta modulator closed-loop force-feedback control schemes have become very attractive<sup>[2]</sup>. It can achieve a direct digital output and can be interactive to a digital processor without other components; it also relaxes the design of analog circuits.

Most of the previous work takes the mechanical part as an integrator to form a second-order single-loop electromechanical Sigma Delta modulator<sup>[3]</sup>. For the second-order electromechanical system, the equivalent DC gain of the mechanical integrators is considerably lower than that of their electronic counterparts, which leads to a much lower signal-to-quantization noise ratio. And limited cycles caused by truncation in elec-

tronic modulators also exist in electromechanical system, which needs to properly compensate the system to prevent limited cycles from moving to the low frequency or degrading the performance. In order to increase the resolution and improve the performance further, some methods in pure electronic system have been introduced to the electromechanical system, such as high order system<sup>[4]</sup>, multistage noise shaping<sup>[5]</sup> and multi-bit quantization. The high signal-to-quantization noise ratio can be obtained by the high order system. However, the stability is still an open issue for the high order system, and the input range decreases as the order increases. Accuracy noise cancellation is difficult to achieve in microfabrication so that multistage noise shaping may be unrealizable in practice. The multi-bit feedback eases the stability problem, reduces the quantization noise, and makes the system more like a linear system, but realizing linear multi-bit feedback in electromechanical system is more complicated than that in electronic sigma delta modulator due to the nonlinear nature of the feedback actuators.

Unlike traditional converters, the over sampling and noise shaping techniques employed in Sigma Delta modulation allow to trade the speed for the accuracy<sup>[6]</sup>. In this paper, a comparative study of Sigma Delta modulation (SDM) and Pulse Code Modulation (PCM) in micro-accelerometers is presented.

## 2 Principle of accelerometer with Sigma Delta modulation and pulse code modulation

A block diagram of the single-loop SDM is shown in Fig 1. It contains three main parts, a B-bit quantizer is used for analog to digital conversion of the integrator output, the output dig-

ital signal is fed back to the input summing node through a D/A conversion, and integrators perform the noise shaping function.

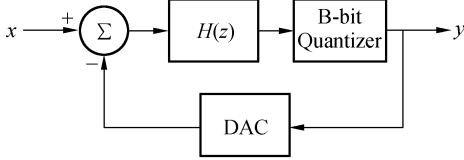


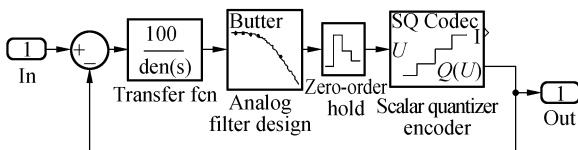
Fig. 1 Single-loop Sigma Delta Modulator

The feedback loop operates at a much higher frequency than the Nyquist frequency of the input signal, which introduces over sampling to reduce noise and improve the system performance. Usually, the digital output is interactive to a digital signal processor and can be constructed at the Nyquist frequency. The mechanical part of an electromechanical system performs as a second-order integrator which is one stage of the overall integrator formed by both the mechanical part and the electronic part or the mechanical part only. PCM schemes are probably the simplest approach to quantizing finite frame expansions. The  $2K$ -level PCM scheme with step size  $\delta$  replaces each  $x_n$  with

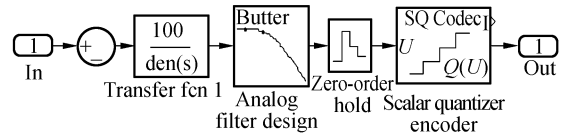
$$q_n = Q(x_n) = \arg \min_{q \in A_K^\delta} |x_n - q|, \quad (1)$$

where  $A_K^\delta = \{(-K + 1/2)\delta, (-K + 3/2)\delta, \dots, (K + 1/2)\delta\}$ .

The architectures of accelerometer with SDM and PCM are shown in Fig. 2, respectively. In addition, both of them are constituted of a 2-bit quantizer. It is implied from Fig. 2 that accelerometer with SDM is a close loop system but accelerometer with PCM is an open loop system.



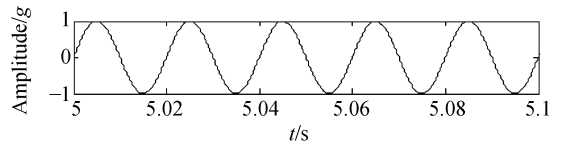
(a) Micro-accelerometer with SDM



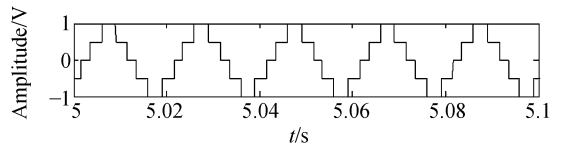
(b) Micro-accelerometer with PCM

Fig. 2 Comparison of architectures of micro-accelerometers with SDM and PCM

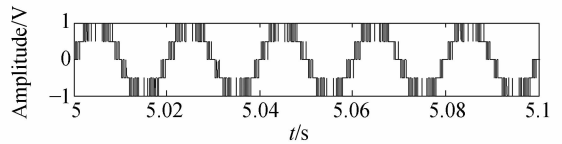
The comparative results of the modulated signals with PCM and SDM are shown in Fig. 3.



(a) Sine wave input



(b) Modulated signal with PCM



(c) Modulated signal with SDM

Fig. 3 Comparison of modulated signals of micro-accelerometers with PCM and SDM

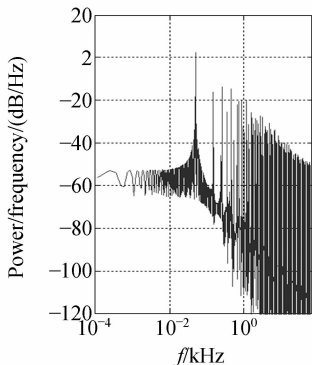
### 3 Power spectral density analysis of SDM and PCM

One of the important applications of signal processing is the spectral analysis. Because of the computational efficiency of FFT, many techniques for spectral analysis of continuous-time or discrete-time signals utilize the DFT either directly or indirectly. There are two basic approaches to estimate the power spectrum. One ap-

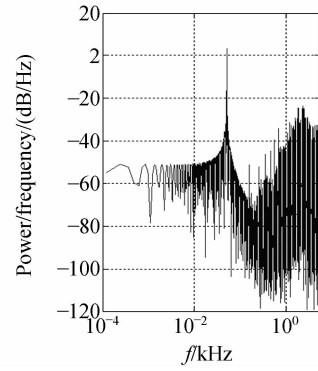
proach is referred to as periodogram analysis and is based on the direct Fourier transformation of finite-length segment of the signals. The other approach firstly estimates the autocovariance sequence and then computes the Fourier transform of this estimation. For the periodogram, if properly normalized, the (modified) periodogram is asymptotically unbiased. Furthermore, the bias approaches zero as the window length increases<sup>[7]</sup>.

The power spectrum densities of the two modulated signals are compared in Fig. 4. It can be clearly seen that the magnitude of quantized noise of SDM is much lower than that of PCM in low frequency, while the magnitudes of quantized noise of the two modulations are almost the same in the high frequency. According to the additive white noise approximation of the quantization error, the quantized noise with PCM is supposed to be white noise, so the magnitude of quantized noise with PCM is uniform in all frequency. However, because of the quantized noise shaping effect of SDM, the quantized noise is shaping by a high pass filter. Thereby, the quantized noise with SDM is much lower than that with PCM in low frequency.

For the sake of demodulation, the modulated signals are filtered by a low pass filter. As Fig. 5 shown, the signal to noise ratio (SNR) of demodulated signal with SDM is higher than that with PCM by about 20 dB.

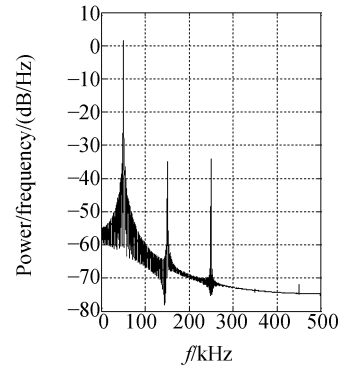


(a) Power spectrum density with PCM

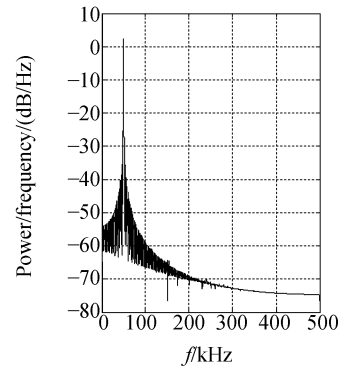


(b) Power spectrum density with SDM

Fig. 4 Comparison of power spectrum densities of modulated signals with PCM and SDM



(a) Spectrum density of demodulated signal with PCM



(b) Spectrum density of demodulated signal with SDM

Fig. 5 Comparison of power spectral densities of demodulated signals

### 4 Conclusions

Comparing the SDM with the PCM, it is implied that the bandwidth of PCM is much wider than that of SDM, as the SDM trades bandwidth for

high resolution, the SDM is not suitable for a high frequency application. However, in the application of measuring low frequency acceleration, such as seismic wave, the SDM is much more appropriate than PCM, as SDM can achieve much higher resolution than PCM, though the

quantizers of both modulations have the same bit width. In addition, because of using a noise shaping technology, the SDM can remove most of the intrinsically nonlinear factor in integrated circuits. As a result, SDM is very suitable for modern MEMS micro-accelerometers.

### References:

- [1] AMINI B V, AYAZI F A. 2.5-V 14-bit  $\Sigma\Delta$  CMOS SOI capacitive accelerometer [J]. *IEEE Journal of Solid-State Circuits*, 2004, 39:2467-2476.
- [2] DONG Y, KRAFT M. Higher order noise-shaping filters for high-performance micromachined accelerometers [J]. *IEEE Transactions on Instrumentation and Measurement*, 2007, 56(5):1666-1674.
- [3] LEMKIN M A, BOSER B E. A Three-axis micromachined accelerometer with a CMOS position-sense interface and digital offset-trim electronics [J]. *IEEE J. Solid-State Circuits*, 1999, 34(4): 456-468.

- [4] AMINI B V, ABDOLVAND R, AYAZI F. A 4.5-mW closed-loop  $\Delta\Sigma$  micro-gravity CMOS SOI accelerometer [J]. *IEEE Journal of Solid-State Circuits*, 2006, 41(12):2983-2991.
- [5] KRAFT M. Closed loop micromachined inertial sensors with higher order  $\Sigma\Delta$  modulators [C]. *Proc. 4th Conf. Model. Simul. Microsyst*, 2001: 104-107.
- [6] RIO R D, MEDEIRO F. *CMOS Cascade Sigma-Delta Modulators for Sensors and Telecom* [M]. Springer: New York, 2006:20-26.
- [7] OPPENHEIM A V, SCHAFER R W, BUCK J R. *Discrete-Time Signal Processing* [M]. Prentice-Hall: New York, 2004:730-755.

### Authors' biographies:



**TAN Xiao-yun** (1960—), male, associate professor of the MEMS Center, Harbin Institute of Technology, his researches focus on MEMS technology, CMOS analog circuit design, etc.. **E-mail:** txj@hit.edu.cn

**Xianzhong Zhou** (1985—), male, M. S.. candidate of the MEMS Center, Harbin Institute of Technology, his researches focus on MEMS technology, CMOS analog circuit design, etc.. **E-mail:** xianzhong1985@163.com

**JIANG Yi-ming** (1983—), female, M. S. candidate of the MEMS Center, Harbin Institute of Technology, her researches focus on MEMS technology, CMOS analog circuit design, etc.. **E-mail:** jiangyiming007@sina.com

**CHEN Wei-ping** (1966—), male, associate professor of the MEMS Center, Harbin Institute of Technology, his research focuses on MEMS technology. **E-mail:** chenwp@hit.edu.cn